ECL 4340

POWER SYSTEMS

LECTURE 11
TRANSMISSION LINE REACTIVE COMPENSATION, Y-BUS MATRIX, POWER FLOWS

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ANNOUNCEMENTS

- Read Chapter 2.4, Network Equations.
- Read Chapter 6.
- HW #6 due October 14, Friday, in Canvas.

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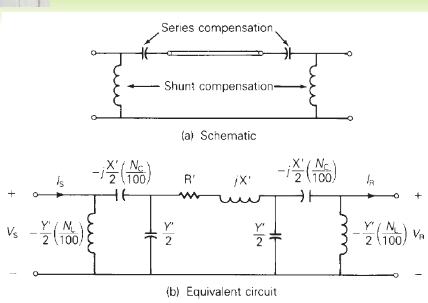
REACTIVE COMPENSATION



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Reactive Compensation

7. Reactive Compensation

Inductors and capacitors are used to increase line loadability and to maintain voltages near rated values.

Shunt reactors (inductors), used in EHV lines
- Reduce overvoltage during light load
- Reduce transient overvoltage due to switching
- Need to be removed during full load

Shunt capacitors
- Increase voltages during heavy load

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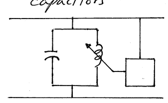
Reactive Compensation

Static Var Compensators (SVC)
- Thyristor-switched reactors with parallel capacitors



Synchronous condensors
- Synchronous motor with no load

Series capacitors
- Reduce net series impedance – reduce line-voltage drop, increase the steady-state stability limit
- subsynchronous resonance – damage turbine-generator shafts

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Reactive Compensation

Ex 5.9 300 km line in Ex 5.2

Use shunt reactors to provide 75% compensation during light load conditions.

Reactors are removed during heavy load:
Full load is 1.90 kA at unity pf, 730 kV.

Find:
a. % voltage regulation before compensation
b. Equivalent shunt admittance & series impedance of the compensated line
c. % voltage regulation after compensation

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Reactive Compensation

(Ex 5.2 lines): 765 kV 300 km

$$z = 0.0165 + j0.3306 = 0.3310 \angle 87.14^\circ \ \Omega/\text{km}$$

$$y = j4.674 \times 10^{-6} \ \text{S/km}$$

$$Z_c = \sqrt{\frac{z}{y}} = 266.1 \angle -1.43^\circ \ \Omega$$

$$\gamma l = \sqrt{yz}\, l = 0.00931 + j0.3730 \ \text{pu}$$

$$A = D = \cosh(\gamma l) = 0.9313 \angle 0.209^\circ \ \text{pu}$$

$$B = 97.0 \angle 87.2^\circ \ \Omega$$

$$C = 1.37 \times 10^{-3} \angle 90.06^\circ \ \text{S}$$

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Reactive Compensation

$$A = D = \cosh(\gamma l) = 0.9313 \angle 0.209^\circ \ \text{pu}$$

$$B = 97.0 \angle 87.2^\circ \ \Omega$$

$$C = 1.37 \times 10^{-3} \angle 90.06^\circ \ \text{S}$$

Reactors are removed during heavy load:
Full load is 1.90 kA at unity pf, 730 kV.

a) % voltage regulation

$$V_S = A\, V_{R_{FL}} + B\, I_{R_{FL}}$$

$$= A\left(\frac{730}{\sqrt{3}} \angle 0^\circ\right) + B\,(1.9 \angle 0^\circ)$$

$$= 442.3 \angle 24.8^\circ \ \text{kV}_{LN}$$

$$V_S = \sqrt{3}\,(442.3) = 766.0 \ \text{kV}_{LL}$$

$$V_{R_{NL}} = \frac{V_S}{A} = \frac{766}{0.9313} = 822.6 \ \text{kV}_{LL}$$

$$\therefore \% VR = \frac{822.6 - 730}{730} \times 100 = \underline{12.68\%}$$

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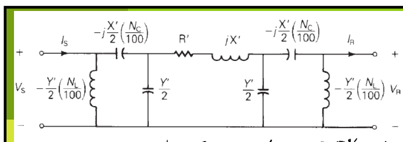

b) Compensation: 75% during light load.

$$\frac{Y'}{2} = \frac{Y}{2}\,\frac{\tanh\frac{\gamma l}{2}}{\frac{\gamma l}{2}} = (7.011 \times 10^{-4} \angle 90^\circ)(1.012 \angle -0.03^\circ)$$

(from Ex. 5.3)

$$= 3.7 \times 10^{-7} + j7.094 \times 10^{-4} \ \text{S}$$

or

$$Y' = 7.4 \times 10^{-7} + j14.188 \times 10^{-4} \ \text{S}$$

With 75% shunt compensation,

$$Y_{eq} = 7.4 \times 10^{-7} + j14.188 \times 10^{-4}\left(1 - \frac{75}{100}\right)$$

$$= 3.547 \times 10^{-4} \angle 89.88^\circ \ \text{S}$$

$$Z_{eq} = Z' = Z\,\frac{\sinh \gamma l}{\gamma l} = (99.3 \angle 87.14^\circ)(0.9969 \angle 0.06^\circ)$$

(from Ex. 5.3)

$$= 97.0 \angle 87.2^\circ \ \Omega$$

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Reactive Compensation

c) Equivalent circuit after compensation:

$$A_{eq} = 1 + \frac{Y_{eq} Z_{eq}}{2} = 1 + \frac{(3.547\times10^{-4})(97.0\angle 87.2^\circ)}{2}$$

$$= 1 + 0.0172\angle 177.1^\circ$$

$$= 0.9828\angle 0.05^\circ \quad pu$$

Then,

$$V_{R_{NL}} = \frac{V_S}{A_{eq}} = \frac{766}{0.9828} = 779.4 \ kV_{LL}$$

$V_{R_{FL}} = 730 \ kV_{LL}$, shunt reactors are removed during heavy load

$$\therefore \% VR = \frac{779.4 - 730}{730} \times 100 = \underline{6.77\%}$$

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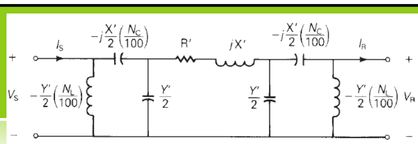
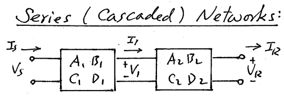


(b) Equivalent circuit

Series (Cascaded) Networks:



$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}, \quad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$= \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$= \begin{bmatrix} A_1A_2 + B_1C_2 & A_1B_2 + B_1D_2 \\ C_1A_2 + D_1C_2 & C_1B_2 + D_1D_2 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

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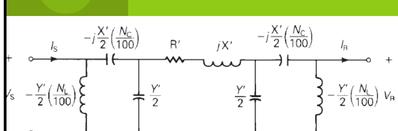
Reactive Compensation



(b) Equivalent circuit

EX. 5.10 Series compensation with 30%
Find theoretical maximum power for the compensated line, $V_S = V_R = 765$ kV

From Ex 5.3, $Z' = 97.0\angle\underline{87.2^\circ}\ \Omega$

$\Rightarrow X' = 97 \sin(87.2^\circ) = 96.88\ \Omega$

30% compensation:

$$Z_{cap} = -jX_{cap} = -j\tfrac{1}{2}(0.3)(96.88) = -j14.53$$

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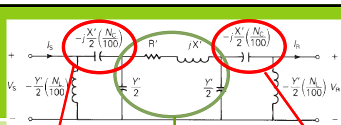

Series networks:

$$\left[\begin{array}{c|c} 1 & -j14.53 \\ \hline 0 & 1 \end{array}\right]\left[\begin{array}{c|c} 0.9313\angle 0.209^\circ & 97.0\angle 87.2^\circ \\ \hline 1.37\times10^{-3}\angle 90.06^\circ & 0.9313\angle 0.209^\circ \end{array}\right]\left[\begin{array}{c|c} 1 & -j14.53 \\ \hline 0 & 1 \end{array}\right]$$

$$\begin{bmatrix} A_{eq} & B_{eq} \\ C_{eq} & D_{eq} \end{bmatrix} = \left[\begin{array}{c|c} 0.9512\angle 0.205^\circ & 69.70\angle 86.02^\circ \\ 1.37\times10^{-3}\angle 90.06^\circ & 0.9512\angle 0.205^\circ \end{array}\right]$$

$$P_{max} = \frac{V_R V_S}{Z'} - \frac{A V_R^2}{Z'}\cos(\theta_B - \theta_A)$$

$$= \frac{765^2}{69.70} - \frac{(0.9512)(765)^2}{69.70}\cos(86.02^\circ - 0.205^\circ)$$

$$= 8396 - 583 = \underline{7813} \text{ MW}$$

: 36.2% larger than 5738 MW (HW 5.5)

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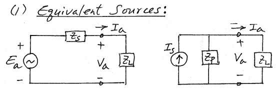
Bus Admittance Matrix or Y_{BUS}

1. Network Matrices (Section 2.4 in Text)

(1) Equivalent Sources:



Voltage Source (Thevenin's)

Current Source (Norton's)

$V_a = E_a - I_a Z_s$ $\quad$ $V_a = (I_s - I_a)Z_p = I_s Z_p - I_a Z_p$

For the two sources to be equivalent,

$E_a = I_s Z_p$ and $Z_s = Z_p$

or $I_s = \frac{E_a}{Z_p} = \frac{E_a}{Z_s}$

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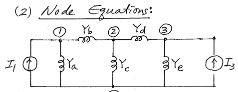
Y_{BUS} Matrix

(2) Node Equations:



Note: Voltage sources are converted to current sources, and impedances are converted to admittances

KCL: $I_1 = Y_a V_1 + Y_b(V_1 - V_2) = (Y_a + Y_b)V_1 - Y_b V_2$

$0 = Y_b(V_2 - V_1) + Y_c V_2 + Y_d(V_2 - V_3)$

$= -Y_b V_1 + (Y_b + Y_c + Y_d)V_2 - Y_d V_3$

$I_3 = Y_d(V_3 - V_2) + Y_e V_3 = -Y_d V_2 + (Y_d + Y_e)V_3$

$\Rightarrow$

(1) $$\begin{bmatrix} I_1 \\ 0 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_a + Y_b & -Y_b & 0 \\ -Y_b & Y_b + Y_c + Y_d & -Y_d \\ 0 & -Y_d & Y_d + Y_e \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

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Y_{BUS} MATRIX

In general, the network equation is

$$I = Y_{bus} V$$

where I is the vector of current injection to buses
V is the vector of bus voltages
Y_{bus} is bus admittance matrix

(3) Building of Y_{bus}:

Y_{ii} (diagonals): self-admittance
= sum of all admittances connected to bus i

$Y_{ij}, i \neq j$ (off-diagonals): mutual-admittance
= negative of the admittance between nodes i and j.

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Y_{BUS} MATRIX

(4) Network Reduction (Kron Reduction):

a Power system has thousands of buses connected and thus the bus admittance matrix is very big in dimension. On the otherhand, there are much fewer number of generators connected to the same network. Therefore, it will be good to reduce the size of the admittance matrix to see the effect of generators on the generator buses only.

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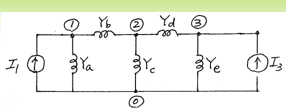
Y_{BUS} MATRIX



For example, the node equation in (1) has one bus (bus #2) without a source connected. To eliminate bus 2, the equations can be rearranged and renumbered as

$$\begin{bmatrix} I_1 \\ I_2 \\ \hline 0 \end{bmatrix} = \left[\begin{array}{cc|c} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ \hline Y_{31} & Y_{32} & Y_{33} \end{array}\right] \begin{bmatrix} V_1 \\ V_2 \\ \hline V_3 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} I_A \\ \hline I_X \end{bmatrix} = \left[\begin{array}{c|c} K & L \\ \hline L^T & M \end{array}\right] \begin{bmatrix} V_A \\ \hline V_X \end{bmatrix}$$

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Y_{BUS} Matrix

$$\begin{bmatrix} I_1 \\ I_2 \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} I_A \\ I_X \end{bmatrix} = \begin{bmatrix} K & L \\ L^T & M \end{bmatrix} \begin{bmatrix} V_A \\ V_X \end{bmatrix}$$

Souling with block matrices,

$$I_A = K V_A + L V_X$$

$$0 = I_X = L^T V_A + M V_X \quad \Rightarrow V_X = -M^{-1} L^T V_A$$

$$\Rightarrow \quad I_A = \underbrace{(K - L M^{-1} L^T)}_{Y_{bus}^{new}} V_A$$

Thus, we now have Y_{bus}^{new} which is of lower dimension than Y_{bus}^{old}.

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Y_{BUS} Matrix

$$\Rightarrow \quad I_A = \underbrace{(K - L M^{-1} L^T)}_{Y_{bus}^{new}} V_A$$

Thus, we now have Y_{bus}^{new} which is of lower dimension than Y_{bus}^{old}.

As a special case, if M is 1×1, scalar, corresponding to bus k to be eliminated,

$$Y_{ij}^{new} = Y_{ij}^{old} - \frac{Y_{ik}^{old} Y_{kj}^{old}}{Y_{kk}^{old}}, \quad i,j = 1, 2, \cdots, n, \quad i,j \neq k$$

We can eliminate one bus at a time, thus avoiding the calculation of the inverse, M^{-1}.

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Power Flow and Y_{BUS} Matrix

- First step in solving the power flow is to create what is known as the bus admittance matrix, often call the $\mathbf{Y}_{bus}$.
- The $\mathbf{Y}_{bus}$ gives the relationships between all the bus current injections, **I**, and all the bus voltages, **V**,
  $\mathbf{I} = \mathbf{Y}_{bus} \mathbf{V}$
- The $\mathbf{Y}_{bus}$ is developed by applying KCL at each bus in the system to relate the bus current injections, the bus voltages, and the branch impedances and admittances

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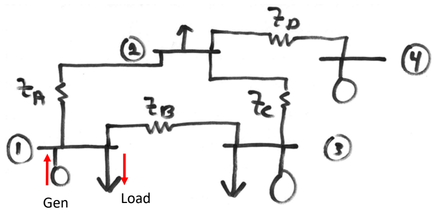
Y_{BUS} Example

Determine the bus admittance matrix for the network shown below, assuming the current injection at each bus i is $I_i = I_{Gi} - I_{Di}$ where I_{Gi} is the current injection into the bus from the generator and I_{Di} is the current flowing into the load



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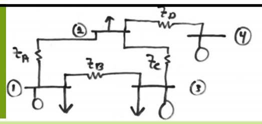

Example, cont'd

By KCL at bus 1 we have

$$I_1 = I_{G1} - I_{D1}$$

$$I_1 = I_{12} + I_{13} = \frac{V_1 - V_2}{Z_A} + \frac{V_1 - V_3}{Z_B}$$

$$I_1 = (V_1 - V_2)Y_A + (V_1 - V_3)Y_B \qquad (\text{with } Y_j = \frac{1}{Z_j})$$

$$= (Y_A + Y_B)V_1 - Y_A V_2 - Y_B V_3$$

Similarly

$$I_2 = I_{21} + I_{23} + I_{24}$$

$$= -Y_A V_1 + (Y_A + Y_C + Y_D)V_2 - Y_C V_3 - Y_D V_4$$

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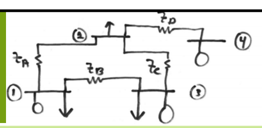

Example, cont'd

We can get similar relationships for buses 3 and 4.
The results can then be expressed in matrix form

$$\mathbf{I} = \mathbf{Y}_{bus}\mathbf{V}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_A + Y_B & -Y_A & -Y_B & 0 \\ -Y_A & Y_A + Y_C + Y_D & -Y_C & -Y_D \\ -Y_B & -Y_C & Y_B + Y_C & 0 \\ 0 & -Y_D & 0 & Y_D \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

For a system with n buses, Y_{bus} is an n by n symmetric matrix (i.e., one where $Y_{ij} = Y_{ji}$)

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Y_{BUS} General Form

- The diagonal terms, Y_{ii}, are the self-admittance terms, equal to the sum of the admittances of all devices incident to bus i.
- The off-diagonal terms, Y_{ij}, are equal to the negative of the sum of the admittances joining the two buses.
- With large systems Y_{bus} is a sparse matrix (that is, most entries are zero)
- Shunt terms, such as with the π-line model, only affect the diagonal terms.

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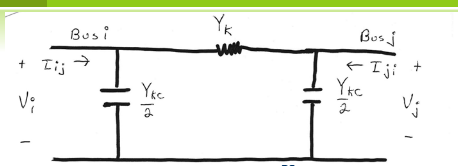
Modeling Shunts in the Y_{BUS}



$$\text{Since } I_{ij} = (V_i - V_j)Y_k + V_i \frac{Y_{kc}}{2}$$

$$Y_{ii} = Y_{ii}^{\text{from other lines}} + Y_k + \frac{Y_{kc}}{2}$$

$$\text{Note } Y_k = \frac{1}{Z_k} = \frac{1}{R_k + jX_k}\frac{R_k - jX_k}{R_k - jX_k} = \frac{R_k - jX_k}{R_k^2 + X_k^2}$$

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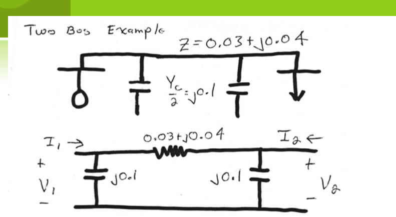
Two Bus System Example



$$I_1 = \frac{(V_1 - V_2)}{Z} + V_1 \frac{Y_c}{2} \qquad \frac{1}{Z} = \frac{1}{0.03 + j0.04} = 12 - j16$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

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Using the Y_{BUS}

If the voltages are known then we can solve for the current injections:

$$\mathbf{Y}_{bus}\mathbf{V} = \mathbf{I}$$

If the current injections are known then we can solve for the voltages:

$$\mathbf{Y}_{bus}^{-1}\mathbf{I} = \mathbf{V} = \mathbf{Z}_{bus}\mathbf{I}$$

where $\mathbf{Z}_{\text{bus}}$ is the *bus impedance* matrix

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Solving for Bus Currents

For example, in previous case assume

$$\mathbf{V} = \begin{bmatrix} 1.0 \\ 0.8 - j0.2 \end{bmatrix}$$

Then

$$\begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.8 - j0.2 \end{bmatrix} = \begin{bmatrix} 5.60 - j0.70 \\ -5.58 + j0.88 \end{bmatrix}$$

Therefore the power injected at bus 1 is

$$\mathrm{S}_1 = V_1 I_1^* = 1.0 \times (5.60 + j0.70) = 5.60 + j0.70$$

$$S_2 = V_2 I_2^* = (0.8 - j0.2) \times (-5.58 - j0.88) = -4.64 + j0.41$$

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Solving for Bus Voltages

For example, in previous case assume

$$\mathbf{I} = \begin{bmatrix} 5.0 \\ -4.8 \end{bmatrix}$$

Then

$$\begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix}^{-1} \begin{bmatrix} 5.0 \\ -4.8 \end{bmatrix} = \begin{bmatrix} 0.0738 - j0.902 \\ -0.0738 - j1.098 \end{bmatrix}$$

Therefore the power injected is

$$\mathrm{S}_1 = V_1 I_1^* = (0.0738 - j0.902) \times 5 = 0.37 - j4.51$$

$$S_2 = V_2 I_2^* = (-0.0738 - j1.098) \times (-4.8) = 0.35 + j5.27$$

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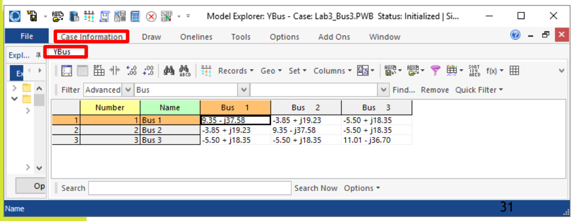
Y_{BUS} IN POWERWORLD

- To see the Y_{bus} in PowerWorld, select Case Information, Solution Details, Ybus
- For large systems most of the Y_{bus} elements are zero, giving what is known as a sparse matrix



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